On collective decisions with interdependent values

Alexandra KIEL*

University of Mannheim

October 2001

*Correspondence address: Alexandra Kiel, LS Güner, Department of Economics, University of Mannheim, L7 3-5, D-68131 Mannheim, Germany; email: akiel@econ.uni-mannheim.de.

Abstract: Many collective decision problems have the common feature that individuals' desired outcomes are correlated but not identical. This paper studies collective decisions with private information about desired policies. Each agent holds private information which mainly concerns his own bliss point, but this private information also affects all other agents. We concentrate on two specific mechanisms, the mean and the median mechanism and establish existence of two symmetric Bayesian Nash equilibria of the corresponding games. Applications of this framework include the assignment of voting rights in the European Central Bank and the design of decision processes in firms or international organizations.

1 Introduction

1.1 Decisions with public values

Many collective decision problems have in common that there is some agreement between the individuals who are supposed to take decisions as well as some disagreement. Our paper models such situations in an environment of asymmetric information by including interdependencies between individual preferences. This means that the individually preferred decision of a group member does not only depend on his own private information but also on the other members' private information. The questions are how the decision mechanism should be designed and how existing mechanisms perform when individual preferences are correlated.

We consider a specific class of collective decision problems where each of these has the following properties: In order to take a common decision, all agents obtain private information (a signal) about their most desired policy. However, no individual is perfectly informed about what would be the privately optimal policy. This imperfection is due to spillover effects between the desired policies. The information of all individuals could be used to calculate the private bliss points whereby each individuals' private information yields more information about its own bliss point than any other individual's private information. Decision problems are characterized by one single parameter which measures the extent to which private information affects all individuals.

In this setting we analyze a specific class of mechanisms. Participation is not voluntary, therefore we can ignore any individual rationality constraints. Moreover, our mechanisms do not condition monetary transfers on the agents' announcements, in fact all monetary transfers are ruled out a-priori. Instead, the mechanisms map individual announcements of the private information into the collective decision.

We concentrate on two mechanisms, i.e. the median and the mean mechanism. The median mechanism implements the median announcement, whereas the mean mechanism implements the average of all announcements. The main difference between these two mechanisms is how they deal with the announcements of private information. Under the median mechanism changes in extreme positions are disregarded, since the median alone determines the final decision. On the contrary, the nature of the mean mechanism is to take all available information into account. Therefore, under the mean mechanism extreme positions influence the decision.

The main result of this paper is the identification of two symmetric Bayesian Nash equilibria of the respective games. Under the median mechanism individuals understate their private information, whereas under the mean mechanism they overstate it. The performance of the mechanisms depends upon the extent to which spillover effects affect the economy.

1.2 Some applications

Many collective decision problems are characterized by a correlation of individual interests. One can think of a number of different applications of our framework:

Any setting in which the individually preferred decision does not only depend on the agents' own private information but as well on the signals of the others fits well into this framework.

One important example may be the decision process in a common central bank like the European Central Bank (ECB). Here, national central banks may care about a policy that accommodates macroeconomic shocks in their own country while taking a collective decision about common monetary policy. However, due to demand spillover effects, shocks in one country may affect the desired policy in other participating countries. Moreover, it is likely that national central bankers have some private information about their national macroeconomic conditions. If interdependencies are strong, the other central bankers' information is very important for the nationally desired policy. These aspects are the more important the closer any EU-enlargement, because on this occasion a discussion about structure and organization of the ESCB, the ECB and its council will become unavoidable to guarantee future functioning. Is the current design appropriate for the degree of economic spillovers that affect the member states and will it remain appropriate after an enlargement of the Union?

The existing literature on decision making in this context focuses either on monetary stabilization policies comparing alternative types of appointees (i.e. having different mandates) in the ECB council (e.g. Von Hagen and Süppel [1994]), on the implications of different policy objectives of the common central bank (e.g. Gros and Hefeker [2000] and Grüner [1999]) or on equilibrium incentive contracts in a multi-principal agency framework (e.g. Dixit and Jensen [2000]). Instead, we ask how the decision mechanism of the ECB (inflation as a function of announced shocks) should be designed in order to maximize the sum of (expected) utilities.

Besides decision making in the ECB council, our setting can be applied for example to international decisions about environmental policy. Basically, the nation states are interested in achieving less pollution in their own country. On the other hand, they have to take a common decision about certain environmental standards. In addition, it may be the case that national governments posses private information about the national amount of emissions, the costs to reduce emissions or the economic consequences of a reduction. However, the environmental situation in one member state is co-determined by the emissions in the neighboring countries. The nearer the location of countries, the more important becomes private information obtained in any single country. Take as a recent example the negotiations about a common European standard of reduction in CO_2 emissions in preparation for the Kyoto Protocol.

Collective decision problems having the features described above can be found also in many areas besides politics. Consider the following example taken from industrial organization: a decision about the future orientation of a firm has to be made. This decision has to be taken by the different heads of department. First of all, these heads are interested in the performance of their own department. Beside this, they posses specific knowledge about the conditions, needs or prospects of it. However, their opinion about the future development of the firm is influenced by the conditions obtaining in other departments as well.

1.3 Related literature

The mechanism design literature offers solutions to related problems, but none to our specific setting. If there are no spillovers and side-payments are allowed it is always possible to obtain (Bayesian) incentive-compatibility using an expected externality mechanism.[1] If informational and allocational externalities are considered but monetary transfers are allowed, the problem of efficient design has been analyzed in auction environments.[2] Instead, we study the case where spillovers are present and monetary transfers are excluded a priori.

Concerning the analysis of collective decisions, the setting of our paper builds an intermediate case between two frameworks commonly used in the literature: On one hand, political outcomes under individual utility maximization are analyzed, i.e. the case of zero spillovers (see Vaubel and Willet [1991] and the references therein). On the other hand, the literature deals with efficient aggregation of perfectly coordinated interests, i.e. 100% spillovers (see Piketty [1999] and the references therein). Our research focuses on the intermediate case: what kind of political outcomes under different information aggregation mechanisms are to be expected if individual interests are correlated to a certain extend? Thus, we do not analyze political outcomes for a fixed degree of spillovers, but instead vary the extent to which individual preferences influence each other.

[1] See Mas-Colell, Whinston and Green [1995] and Arrow [1979] or D'Aspremont and Gérard-Varet [1979].

[2] See for example Fieseler, Kittsteiner and Moldovanu [2000] or Jehiel and Moldovanu [1998].

Related to our work is a recent paper by Casella [2000]. In a similar informational environment but with private values she proposes a simple voting scheme for deliberations taken by committees that meet regularly over time. At each meeting, committee members are allowed to store their vote for future use. Although the scheme cannot achieve the first best with more than two voters, making votes storable typically leads to ex ante welfare gains. Her paper differs from ours in that we do not consider developments over time. Instead, we study a one-shot game excluding also any reputation effects a priori.

The remainder of this paper is organized as follows: Section 2 introduces the model presenting the first-best solution and the two mechanisms we aim to study. The respective equilibria under those mechanisms are calculated in Section 3 where results about the truthful revelation properties of the equilibria are given as well. Section 4 concludes. All omitted proofs can be found in the Appendix.

2 The Model

We consider an economy which is populated by $i = 1, 2, 3$ individuals. A decision $x \in \mathbb{R}$ has to be taken. Each individual receives private information θ_i. The parameters θ_i are distributed independently and uniformly over the support $[-1, 1]$. The vector of private information is θ. An individual's preferences over outcomes are characterized by the following von-Neumann-Morgenstern utility function

$$u_i(x, \theta) = -\left(x - \theta_i^*\right)^2 \tag{1}$$

where θ_i^* describes the individually preferred decision. Individual utility is maximized at $x = \theta_i^*$ (i.e. when the individually preferred decision is actually implemented). The highest attainable utility is zero. The larger the difference between the implemented policy x and the individually preferred decision θ_i^*, the smaller is individual utility. The individually preferred policy θ_i^* is a convex combination of $i's$ type and the average of all others' types, i.e.

$$\theta_i^* = (1 - \alpha)\,\theta_i + \frac{\alpha}{2}\sum_{j \neq i} \theta_j. \qquad (2)$$

The parameter $\alpha \in [0, \bar{\alpha}]$ measures the extent to which interdependencies align the preferences of all individuals. For the upper bound value $\bar{\alpha} := \frac{2}{3}$ all individuals share a common utility function with a maximum at $x = \theta_{Mean}$ where $\theta_{Mean} = \sum_{i=1}^{3} \theta_i \,/\, 3$. For the lower bound $\alpha = 0$ each individual has his signal θ_i as a private bliss point. For example, α might measure the degree of demand spillover effects between the members of the European Union or the degree to which firm departments are interlinked.

2.1 Welfare

First, we describe the decision that maximizes welfare, when there are no informational asymmetries. Welfare is defined as the sum of individual utilities, i.e. we take an utilitarian perspective to welfare. It turns out this decision x^* is independent of the degree of interdependency and is determined by the average of all private signals.

Lemma 1 *The sum of all utilities is maximized at* $x^* = \theta_{Mean}$.

PROOF: See Appendix.

The intuition for this result is that by summing up the utilities of all individuals any spillover effects are automatically taken into account. Lemma 1 implies that the mean mechanism would yield the first best if truth-telling could be implemented for all degrees of interdependency. However, if informational asymmetries are present, this first-best solution remains no longer attainable.

2.2 The mechanisms

We consider the following two direct mechanisms excluding any monetary transfers and ignoring participation constraints: All individuals are asked for an announcement $\hat{\theta}_i \in \mathbb{R}$ of their private signal. The vector of announcements is $\hat{\theta}$. Depending on these announcements the collective decision x is taken.

The first mechanism we study is the median mechanism. Let $x_{Median} = x_{Median}(\hat{\theta}) := \hat{\theta}_{Median}$, where $\hat{\theta}_{Median}$ is the median of all announcements. Let θ_{Median} denote the signal obtained by the median individual. Note that in general $\theta_{Median} = \hat{\theta}_{Median}$ is not true. The median mechanism implements the median of all announcements and thus replicates majority decisions for the case of zero spillovers. Another feature of this mechanism is that changes in extreme positions are disregarded, since the final decision solely depends on the median announcement.

The second mechanism we analyze is the mean mechanism. Let $x_{Mean} = x_{Mean}(\hat{\theta}) := \frac{1}{3}\sum_{i=1}^{3} \hat{\theta}_i(\theta_i)$. This mechanism asks the individuals for their private information and implements the average of all announcements. Consequently, the mean mechanism uses all available information. Thus extreme positions influence the common decision.

3 The Results

In this section, we present two Bayesian Nash equilibria of the games introduced above, one of the median and one of the mean mechanism. These equilibria imply truthful revelation for certain degrees of interdependencies.

3.1 Equilibria

Proposition 1 *The median mechanism has a symmetric Bayesian Nash equilibrium. The equilibrium strategy is* $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i$.

Proof: See Appendix.

Under the median mechanism individuals understate their private information. With increasing degrees of interdependency (larger α) the private signal becomes less valuable. Since individuals know that the median of all announcements will be implemented, they try to profit from the others' information. Their announcement is closer to zero than their true signal due to the fact that zero is the expected value of the information parameters.

Proposition 2 *i) There exists a continuum of other symmetric equilibria under the median mechanism in which all individuals announce the same type irrespective of their signal, i.e.* $\hat{\theta}_i(\theta_i) = \widetilde{\theta}\ \forall i$. *ii) All those equilibria yield lesser expected utility for all individuals than* $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i$.

Proof: See Appendix.

In the following, we concentrate on equilibrium strategies $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i$ $\forall i$, because this equilibrium yields at least the same expected payoff than the other equilibria.

We obtain another linear equilibrium strategy for the mean mechanism. According to this strategy individuals overstate their private information.

Proposition 3 *The mean mechanism has a symmetric Bayesian Nash equilibrium. The equilibrium strategy is* $\hat{\theta}_i(\theta_i) = 3\,(1-\alpha)\,\theta_i$.

PROOF: See Appendix.

Under the mean mechanism individuals exaggerate their private signal in order to cancel out the average taking implied by the mean mechanism. With increasing degrees of interdependency this behavior becomes counterproductive and announcements approach the true signal. Note that this result extends to the case of n individuals. Then, the equilibrium strategy under the mean mechanism is given by $\hat{\theta}_i(\theta_i) = n\,(1-\alpha)\,\theta_i$.

Although it may seem at first sight that both equilibrium strategies are independent of the underlying distribution of information parameters, this indeed is not the case and due to the specification of individual utility.

Using the above calculated equilibrium strategies, it follows that

Corollary 1 *The median mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if* $\alpha = 0$.

Corollary 2 *The mean mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if* $\alpha = \bar{\alpha} = \frac{2}{3}$.

4 Conclusion

The problem analyzed in this paper is one of collective decision taking. If the individuals who are supposed to take a common decision are asymmetrically informed and if there are interdependencies between the individually desired policies, how should a decision mechanism be designed that maximizes the sum of expected utilities? Our analysis of this problem concentrates on two specific mechanisms, i.e. the median and the mean mechanism, and obtains the following: Under the median mechanism individuals understate their private information, whereas under the mean mechanism they overstate it. The performance of the respective mechanisms depends upon the extent to which spillover effects affect the economy.

After analyzing equilibrium behavior under the median and the mean mechanism for varying degrees of interdependencies, the next step will be to compare the performance of these mechanisms for different degrees of correlation between individual preferences. The final aim would be a clear-cut prediction as to when which mechanism yields the better result in terms of expected utility maximization. Considering the obtained equilibrium strategies, we suppose that the median mechanism will outperform the mean mechanism for a wide range of low interdependencies.

Starting from this research there are four directions to proceed. The first extension is the analysis for larger n. For the mean mechanism all results obtained hold for $n > 3$, but for the median mechanism this is not obvious. Once there are results for the median mechanism as well, one could introduce a class of mechanisms which relates the outcome to the average of some agents' announcements about their private information. This class includes the median and the mean mechanism as special cases.

Second, in this paper we abstracted from any individual rationality considerations, since in many collective decisions participation is not voluntary. However, if we take participation constraints into account, the traditional solution would prescribe an outside option to be implemented when an individual opts out. This in turn leads to changed interim individual behavior and to different equilibrium outcomes - with the status quo maintaining in many instances. But in our setting - due to interdependent valuations - even individuals not participating in the mechanism would be affected by the collective decision. This would imply that one has to endogenize the participation constraint (following Jehiel, Moldovanu and Stacchetti [1996]).

Third, in a modified two-stage game the issue of pre-vote communication may be analyzed. The question is if an improvement upon the equilibria of the original game is possible when people are allowed to communicate before they have to vote. It is well known that equilibrium behavior can be affected if agents have the opportunity to exchange information prior to playing some game (see Crawford and Sobel [1982]). Our intuition is that such an improvement is not possible in our framework.

Finally, another question we did not address is the design of an optimal mechanism for the class of collective decision problems studied. This would mean to find a mechanism that implements the first-best for all degrees of spillovers, not only for the maximum amount.

5 Appendix - Proofs

Proof of Lemma 1:

The sum of all utilities is maximized if x^* maximizes

$$\sum_{i=1}^{3} u_i(x,\theta) = \sum_{i=1}^{3} -(x-\theta_i^*)^2. \tag{3}$$

Optimality requires

$$-\sum_{i=1}^{3} 2(x^*-\theta_i^*) = 0 \tag{4}$$

$\iff$

$$\begin{aligned} x^* &= \frac{1}{3}\sum_{i=1}^{3}\theta_i^* \qquad (5)\\ &= \frac{1}{3}\sum_{i=1}^{3}\left((1-\alpha)\theta_i + \frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)\\ &= \frac{1}{3}\left((1-\alpha)\sum_{i=1}^{3}\theta_i + \frac{\alpha}{2}\sum_{i=1}^{3}\sum_{j\neq i}\theta_j\right)\\ &= \frac{1}{3}\sum_{i=1}^{3}\theta_i\\ &= \theta_{Mean}. \end{aligned}$$

Q.E.D.

Proof of Proposition 1:

Without loss of generality consider individual 2's best response to linear equilibrium strategies $\hat{\theta}_i(\theta_i) = a\theta_i,\ i = 1,3$. Individual 2 maximizes its expected utility under the median mechanism if $\hat{\theta}_2(\theta_2)$ maximizes

$$E\left(\hat{\theta}_2\right) := E\left[-\left(x_{Median}-\theta_2^*\right)^2\right]. \tag{6}$$

This expected utility can be decomposed into three parts in the following way: $E\left(\hat{\theta}_2\right) = E_1\left(\hat{\theta}_2\right) + E_2\left(\hat{\theta}_2\right) + E_3\left(\hat{\theta}_2\right)$. The first part $E_1\left(\hat{\theta}_2\right)$ describes the situation in which the announcement of individual 2 is neither the highest nor the lowest announcement, i.e. $\hat{\theta}_2 = \hat{\theta}_{Median}$. Individual 2's announcement is then implemented according to the mechanism, i.e. $x_{Median} = \hat{\theta}_2$. The second part $E_2\left(\hat{\theta}_2\right)$ describes the case in which the announcements of the two other agents are both below the announcement of individual 2, and the third $E_3\left(\hat{\theta}_2\right)$ the case in which they are both above, respectively. We analyze these three cases in turn.

Consider first the situation in which the announcement of individual 2 is the median announcement. This happens if either $\hat{\theta}_1 < \hat{\theta}_2 < \hat{\theta}_3$ or $\hat{\theta}_3 < \hat{\theta}_2 < \hat{\theta}_1$. Since these two cases are symmetric concerning individual 2's expected utility, assume without loss of generality $\hat{\theta}_1 < \hat{\theta}_2 < \hat{\theta}_3$ and multiply the resulting part of expected utility by 2. The corresponding component is then given by

$$E_1\left(\hat{\theta}_2\right) = -\frac{1}{2}\int_{-1}^{\frac{\hat{\theta}_2}{a}}\int_{\frac{\hat{\theta}_2}{a}}^{1}\left(\hat{\theta}_2 - (1-\alpha)\,\theta_2 - \frac{\alpha}{2}\,(\theta_1+\theta_3)\right)^2 d\theta_3 d\theta_1. \tag{7}$$

Next, take the case in which the announcements of both other agents are below the announcement of individual 2, i.e. if either $\hat{\theta}_1 < \hat{\theta}_3 < \hat{\theta}_2$ or $\hat{\theta}_3 < \hat{\theta}_1 < \hat{\theta}_2$. Again, these two cases are symmetric. Assume without loss of generality $\hat{\theta}_1 < \hat{\theta}_3 < \hat{\theta}_2$ and multiply the resulting part by 2. This situation yields

$$E_2\left(\hat{\theta}_2\right) = -\frac{1}{2}\int_{-1}^{\frac{\hat{\theta}_2}{a}}\int_{\theta_1}^{\frac{\hat{\theta}_2}{a}}\left(a\theta_3 - (1-\alpha)\,\theta_2 - \frac{\alpha}{2}\,(\theta_1+\theta_3)\right)^2 d\theta_3 d\theta_1. \tag{8}$$

Finally, consider the situation in which the announcements of the two other agents are both above the announcement of individual 2, i.e. if either $\hat{\theta}_2 < \hat{\theta}_1 < \hat{\theta}_3$ or $\hat{\theta}_2 < \hat{\theta}_3 < \hat{\theta}_1$. These two cases are symmetric as well. Thus, assume without loss of generality $\hat{\theta}_2 < \hat{\theta}_1 < \hat{\theta}_3$ and multiply the resulting part by 2. The corresponding component of expected utility is given by

$$E_3\left(\hat{\theta}_2\right) = -\frac{1}{2}\int_{\frac{\hat{\theta}_2}{a}}^{1}\int_{\theta_1}^{1}\left(a\theta_1 - (1-\alpha)\,\theta_2 - \frac{\alpha}{2}(\theta_1+\theta_3)\right)^2 d\theta_3 d\theta_1. \tag{9}$$

Taking all three parts together and differentiating with respect to $\hat{\theta}_2$ yields

$$\begin{aligned} &\frac{d\left(E\left[-\left(x_{Median}-\theta_2^*\right)^2\right]\right)}{d\hat{\theta}_2} \\ = &\left(\hat{\theta}_2 - a\right)\left(\hat{\theta}_2 + a\right)\frac{2a\hat{\theta}_2 - \alpha\hat{\theta}_2 + 2\alpha a\theta_2 - 2a\theta_2}{2a^3}. \end{aligned} \tag{10}$$

An interior solution to this maximization problem requires

$$\frac{d\left(E\left[-\left(x_{Median}-\theta_2^*\right)^2\right]\right)}{d\hat{\theta}_2} = 0.$$

The three solutions to this equation are given by

$$\begin{aligned} \hat{\theta}_{2,1} &= -a, \\ \hat{\theta}_{2,2} &= 2a\frac{\alpha-1}{\alpha-2a}\theta_2, \end{aligned} \tag{11}$$

and

$$\hat{\theta}_{2,3} = a.$$

Note that for announcements $\hat{\theta}_2$ above a and below $-a$ expected utility for individual 2 is not longer given by $E_1\left(\hat{\theta}_2\right) + E_2\left(\hat{\theta}_2\right) + E_3\left(\hat{\theta}_2\right)$. In case of individuals

1 and 3 announcing according to $\hat{\theta}_i(\theta_i) = a\theta_i$, the announcement $\hat{\theta}_2$ is never the median announcement for $\hat{\theta}_2 > a$ and $\hat{\theta}_2 < -a$. Then, individual 2's expected utility is constant. For $\hat{\theta}_2 > a$, it is given by

$$E_2(a) = -\frac{1}{2}\int_{-1}^{1}\int_{\theta_1}^{1}\left(a\theta_3 - (1-\alpha)\theta_2 - \frac{\alpha}{2}(\theta_1+\theta_3)\right)^2 d\theta_3 d\theta_1 \tag{12}$$

and for $\hat{\theta}_2 < -a$ by

$$E_3(-a) = -\frac{1}{2}\int_{-1}^{1}\int_{\theta_1}^{1}\left(a\theta_1 - (1-\alpha)\theta_2 - \frac{\alpha}{2}(\theta_1+\theta_3)\right)^2 d\theta_3 d\theta_1. \tag{13}$$

We now return to the three possible equilibria described by (11). Since we are interested in equilibrium strategies that are linear in θ_2 we start the further analysis with concentrating on $\hat{\theta}_{2,2}$. In order to determine the factor a one has to solve

$$a = 2a\frac{\alpha - 1}{\alpha - 2a} \tag{14}$$

which gives the two solutions

$$\begin{aligned} a_1 &= 0 \\ a_2 &= 1 - \frac{1}{2}\alpha. \end{aligned} \tag{15}$$

Again, since we are looking for an equilibrium with linear announcement strategies[3], we concentrate on $a_2 = 1 - \frac{1}{2}\alpha =: \widetilde{a}$. For this specific linear factor two observations can be made:

(i) $\widetilde{a} > 0$ for $\alpha \in [0, \bar{\alpha}]$ and

(ii) $\hat{\theta}_{2,1} \leq \hat{\theta}_{2,2} \leq \hat{\theta}_{2,3}$, since $\hat{\theta}_{2,2} = \left(1 - \frac{1}{2}\alpha\right)\theta_2$ for $\widetilde{a}$ and $\theta_2 \in [-1, 1]$.

[3]Compare also Proposition 2.

It remains to show that $\hat{\theta}_{2,2}$ indeed yields a maximum of individual announcement behavior for $\widetilde{a}$. Consider the first derivative (10) and define

$$f(\hat{\theta}_2) := \frac{2\widetilde{a}\hat{\theta}_2 - \alpha\hat{\theta}_2 + 2\alpha\widetilde{a}\theta_2 - 2\widetilde{a}\theta_2}{2\widetilde{a}^3} \tag{16}$$

and

$$g(\hat{\theta}_2) := \left(\hat{\theta}_2 - \widetilde{a}\right)\left(\hat{\theta}_2 + \widetilde{a}\right). \tag{17}$$

In order to check the second order condition we consider f and g in turn. It holds that

$$f(\hat{\theta}_{2,2}) = 0, \tag{18}$$

$$f'(\hat{\theta}_2) = \frac{2\widetilde{a} - \alpha}{2\widetilde{a}^3} > 0 \tag{19}$$

and for $\alpha \in [0, \bar{\alpha}]$ and $\theta_2 \in [-1, 1]$,

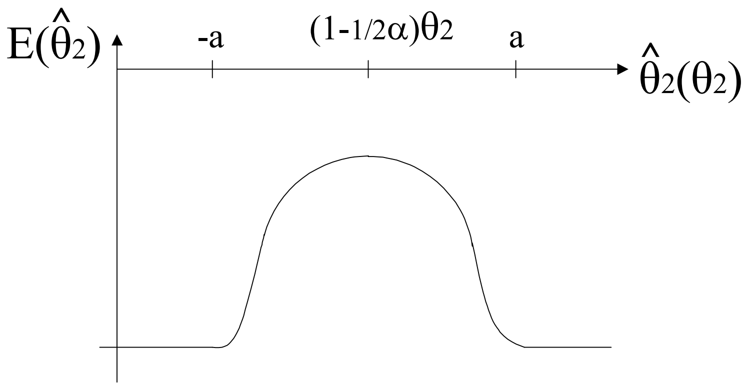
$$g(\hat{\theta}_{2,2}) = \left(\left(1 - \frac{1}{2}\alpha\right)\theta_2\right)^2 - \left(1 - \frac{1}{2}\alpha\right)^2 < 0. \tag{20}$$

The second derivative at $\hat{\theta}_2 = \hat{\theta}_{2,2}$ is given by

$$\left.\frac{d\left(E\left[-\left(x_{Median} - \theta_2^*\right)^2\right]\right)^2}{d^2\hat{\theta}_2}\right|_{\hat{\theta}_2 = \hat{\theta}_{2,2}} \tag{21}$$

$$\begin{aligned} &= g'(\hat{\theta}_{2,2})f(\hat{\theta}_{2,2}) + f'(\hat{\theta}_{2,2})g(\hat{\theta}_{2,2}) \\ &= f'(\hat{\theta}_{2,2})g(\hat{\theta}_{2,2}) \\ &< 0. \end{aligned}$$

Note that a similar argument shows that $\hat{\theta}_{2,1}$ and $\hat{\theta}_{2,3}$ indeed yield minima of individual 2's expected utility for $\widetilde{a}$. The following (simplified) picture shows individual 2's expected utility as a function of its announcement.



Hence, given that individuals 1 and 3 announce according to $\hat{\theta}_i(\theta_i) = \widetilde{a}\theta_i$, $\widetilde{a} = 1 - \frac{1}{2}\alpha$, the strategy

$$\hat{\theta}_2(\theta_2) = \left(1 - \frac{1}{2}\alpha\right)\theta_2 \tag{22}$$

is a best reply. Q.E.D.

PROOF OF PROPOSITION 2:

i) EXISTENCE: Assume without loss of generality that individuals 1 and 2 announce according to $\hat{\theta}_i(\theta_i) = \widetilde{\theta}$, $i = 1, 2$. That being so, it is a best reply for individual 3 to announce $\widetilde{\theta}$ as well, since on no account its announcement will change the implemented decision under the median mechanism.

ii) If agents announce $\hat{\theta}_i(\theta_i) = (1 - \frac{1}{2}\alpha)\theta_i$ then $x_{Median} = (1 - \frac{1}{2}\alpha)\theta_{Median}$. Consider without loss of generality expected utility for individual 2. This is given by

$$E\left[-\left(x_{Median} - \theta_2^*\right)^2\right]$$

$$
\begin{aligned}
&= -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\theta_2}\int_{\theta_2}^{1}\left(\left(1-\frac{1}{2}\alpha\right)\theta_2-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&\quad -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\theta_2}\int_{\theta_1}^{\theta_2}\left(\left(1-\frac{1}{2}\alpha\right)\theta_3-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&\quad -\frac{1}{4}\int_{-1}^{1}\int_{\theta_2}^{1}\int_{\theta_1}^{1}\left(\left(1-\frac{1}{2}\alpha\right)\theta_1-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&= -\frac{11}{20}\alpha^2+\frac{11}{15}\alpha-\frac{4}{15}. \qquad (23)
\end{aligned}
$$

If agents announce $\hat{\theta}_i(\theta_i)=\widetilde{\theta}$ then $x_{Median}=\widetilde{\theta}$. Again, consider expected utility for individual 2:

$$
\begin{aligned}
&E\left[-\left(x_{Median}-\theta_2^*\right)^2\right] \\
&= -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\widetilde{\theta}}\int_{\widetilde{\theta}}^{1}\left(\widetilde{\theta}-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&\quad -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\widetilde{\theta}}\int_{\theta_1}^{\widetilde{\theta}}\left(\widetilde{\theta}-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&\quad -\frac{1}{4}\int_{-1}^{1}\int_{\widetilde{\theta}}^{1}\int_{\theta_1}^{1}\left(\widetilde{\theta}-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
&= -\frac{1}{2}\alpha^2+\frac{2}{3}\alpha-\frac{1}{3}-\widetilde{\theta}^2. \qquad (24)
\end{aligned}
$$

Since $\widetilde{\theta}^2>0$, it suffices to show that

$$
-\frac{1}{2}\alpha^2+\frac{2}{3}\alpha-\frac{1}{3}<-\frac{11}{20}\alpha^2+\frac{11}{15}\alpha-\frac{4}{15} \qquad (25)
$$

which is true $\forall\ \alpha\in[0,\bar{\alpha}]$. Q.E.D.

Proof of Proposition 3:

Consider individual i's best response to the equilibrium strategy. Individual i maximizes its expected utility if $\hat{\theta}_i(\theta_i)$ maximizes

$$E\left[-\left(x_{Mean}-\theta_i^*\right)^2\right]=E\left[-\left(\frac{\sum_{j=1}^3\hat{\theta}_j(\theta_j)}{3}-\theta_i^*\right)^2\right]. \tag{26}$$

Substituting for θ_i^* yields

$$\max_{\hat{\theta}_i(\theta_i)} E\left[-\left(\frac{\sum_{j=1}^3\hat{\theta}_j(\theta_j)}{3}-(1-\alpha)\,\theta_i-\frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)^2\right] \tag{27}$$

$$\max_{\hat{\theta}_i(\theta_i)} E\left[-\left(\frac{\hat{\theta}_i(\theta_i)}{3}-(1-\alpha)\,\theta_i+\frac{\sum_{j=1,\,j\neq i}^3\hat{\theta}_j(\theta_j)}{3}-\frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)^2\right] \tag{28}$$

$$\begin{aligned}\max_{\hat{\theta}_i(\theta_i)}\ & -E\left[\frac{1}{9}\hat{\theta}_i(\theta_i)^2\right] \\ & -2E\left[\left(\frac{1}{3}\hat{\theta}_i(\theta_i)\right)\left(-(1-\alpha)\,\theta_i+\frac{\sum_{j=1,\,j\neq i}^3\hat{\theta}_j(\theta_j)}{3}-\frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)\right] \\ & -E\left[\left(-(1-\alpha)\,\theta_i+\frac{\sum_{j=1,\,j\neq i}^3\hat{\theta}_j(\theta_j)}{3}-\frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)^2\right].\end{aligned} \tag{29}$$

Using the fact that θ_j and $\hat{\theta}_j(\theta_j)$ have an expected value of zero and that expectations are taken over all $j\neq i$, we get

$$\begin{aligned}\max_{\hat{\theta}_i(\theta_i)}\ & -\frac{1}{9}\hat{\theta}_i(\theta_i)^2-2\left(\frac{1}{3}\hat{\theta}_i(\theta_i)\right)\left(-(1-\alpha)\,\theta_i\right) \\ & -E\left[\left(-(1-\alpha)\,\theta_i+\frac{\sum_{j=1,\,j\neq i}^3\hat{\theta}_j(\theta_j)}{3}+\frac{\alpha}{2}\sum_{j\neq i}\theta_j\right)^2\right].\end{aligned} \tag{30}$$

Optimality requires

$$-\frac{2}{9}\hat{\theta}_i(\theta_i) - \frac{2}{3}\left(-\left(1-\alpha\right)\theta_i\right) = 0 \tag{31}$$

and thus,

$$\hat{\theta}_i(\theta_i) = 3\left(1-\alpha\right)\theta_i. \tag{32}$$

Q.E.D.

References

[1979] Arrow, K. (1979) "The property rights doctrine and demand revelation under incomplete information," in *Economics and Human Welfare.* edited by M. Boskin. New York: Academic Press.

[2000] Casella, A. (2000) "Towards a theory of storable votes," Mimeo, Columbia University, New York.

[1982] Crawford, V. and J. Sobel (1982) "Strategic information transmission," *Econometrica*, 50, 1431-1452.

[1979] D'Aspremont, C. and L.A. Gérard-Varet (1979) "Incentives and incomplete information," *Journal of Public Economics,* 11, 25-45.

[2000] Dixit, A. and H. Jensen (2000) "Equilibrium contracts for the central bank of a monetary union," CES Working Paper 400.

[2000] Fieseler, K., T. Kittsteiner and B. Moldovanu (2000) "Partnerships, Lemons and Efficient Trade," Discussion Paper, University of Mannheim.

[2000] Gros, D. and C. Hefeker (2000) "One size must fit all - national divergences in a monetary union," CEPS Working Document 149.

[1999] Grüner, H. P. (1999) "On the role of conflicting national interests in the ECB Council," CEPR Discussion Paper 2192.

[1998] Jehiel, P. and B. Moldovanu (1998) "Efficient Design with Interdependent Valuations," Discussion Paper, Northwestern University, forthcoming in *Econometrica.*

[1996] Jehiel, P., B. Moldovanu and E. Stacchetti (1996) "How (not) to sell nuclear weapons," *American Economic Review*, 86(4), 814-829.

[1995] Mas-Colell, A.,M. D. Whinston and J. R. Green (1995) *Microeconomic Theory.* New York, Oxford: Oxford University Press.

[1999] Piketty, T. (1999) "The information-aggregation approach to political institutions," *European Economic Review*, 43, 791-800.

[1991] Vaubel, R. and T.D. Willet (1991) *The political economy of international organizations: A public choice approach.* Political Economy of Global Interdependence series, Boulder and Oxford: Westview Press.

[1994] Von Hagen, J. and R. Süppel (1994) "Central bank constitutions for federal monetary unions," *European Economic Review,* 38, 774-782.