EPIC, Second Research Workshop, Florence 16 – 21 May 2002

The Role of Exchange Rate in Monetary Policy Rules: An Empirical Study

Jan Strasky, Oxford University

This paper examines in some detail the role of the exchange rate in the design of monetary policy rules. While the rules of Taylor type seem to be optimal for closed economies, the issue of an appropriate policy rule for the open economy is still widely debated. In what follows I mean by the Taylor rule any specification where the interest rate as an instrument of policy is set according to deviations of inflation and output from their long-run equilibrium levels. By complex rule I mean such a specification of the rule which incorporates the exchange rate, either in the form of a Monetary Condition Index, or as another term on the right-hand side of the equation. The paper focuses on the differences between simple and complex rules, and its research question is whether there is a potential gain from complex rules in open economy conditions.

I start in Section 1, discussing briefly the practical relevance of the problem, and some issues in the research design. Then, Section 2 focuses on theoretical issues and gives some arguments in favour of including the exchange rate in the policy rule. This also serves as a preliminary review of literature as far as the macroeconomic theory is concerned. Concluding, Section 3 deals with econometric issues related to the problem. It first discusses VAR and equation systems modelling in stationary set-up. Then, the analysis is extended to the cointegration framework, where we outline the major steps in estimation of a small macroeconomic model that may then serve as a basis for evaluation based on stochastic simulations. Appendices then summarise our results so far and provide detailed information on the model specification I plan to start work with.

1. Relevance for Policy and the Research Design

The issue of an optimal monetary policy rule has important implications for the practical implementation of monetary policy. Several models described in literature are very similar to those used in the decision making process of some central banks (e. g., Bank of England). If there is a welfare gain from stabilising the exchange rate, this would, indeed, be of the utmost importance for policy-makers. The analysis will also shed light on theoretical issues such as inflationary bias, credibility, and time-consistency that are among the main reasons for ongoing research in the area.

The research design is a combination of modelling and an empirical case study. I intend to use the Czech Republic and Polish data as two cases of a small open economy setting. I intend to test whether there is evidence in favour of what I have described above as complex rules. Techniques of policy evaluation are based on Taylor (1993). The model solution incorporating model consistent expectations rests on McCallum (1998) and Uhlig (1995).

Two approaches can be considered. Firstly, the model can be estimated using empirical data, and then, different specifications of policy rules incorporated into the model. The outcomes will be directly compared in terms of their relative inflation and output volatility. This requires a fully specified macro model of the two economies.

Secondly, the outcomes relevant for policy making can be stochastically simulated on the basis of the estimated steady-state variance covariance matrix, or with assumptions about the shock structure imposed by the researcher. This will again yield results that are comparable in terms of the inflation and output volatility and characterise varying values of the policy rule coefficients.

These strategies are both in line with the four research steps that are being used in the area of economic policy modelling and evaluation. These steps, summed up along Taylor (2001), also illustrate nicely the tasks that need to be solved during my research:

1. Place a potential MP rule into a macroeconomic model
2. Solve the model using numerical methods
3. Examine the properties of the stochastic behaviour of variables (inflation, output)
4. Choose the rule that performs most satisfactory with respect to the loss function that is as close to the real world preferences as possible.

The fist step emphasises the need to set up or select a proper theoretical macro model that should be used in analysis. In order to clarify this issue, we now turn to theoretical consideration.

2. Open Economy Theory: Exchange rate, Pass-through, and Choice of Policy

Here, we deal with three related issues in turn. First, we discuss the ways in which the exchange rate changes affect an open economy. Then we provide a theoretically based argument for the inclusion of exchange rate in the monetary policy rule.

Exchange rate effects

The exchange rate enters many models as an important part of the transmission mechanism. The link usually works through the interest rate arbitrage condition that relates domestic interest rate to foreign interest rate and (expected) rate of appreciation of the exchange rate. As many models assume perfect capital mobility, they either incorporate an ex-ante interest rate parity condition (UIP) or a reduced form relationship between a real interest rate and real exchange rate implied by UIP.

The exchange rate also affects the terms of trade and hence flows of exports and imports. This is where the consideration of the ways in which changes in the exchange rate affect prices of imported goods comes into play. Changes in import prices then indeed affect the domestic price level. Choices made in modelling of the pass-through[1] may substantially affect resulting dynamic properties of the model.

As some of the recent evidence suggest (cf. Burstein, Eichenbaum et al. (2001)), the assumption of an immediate pass-through seems to be refuted by empirical data. An immediate pass-through assumption usually involves the full increase in import prices after devaluation being immediately translated into the CPI price level. Under such assumption, the total effect is determined by proportion of imported goods in the CPI. However, post-devaluation inflation levels are much smaller than ones that would be consistent with this story.

Hence, an improved and rather optimising-based modelling of price rigidities ("optimally-derived staggered price setting equations with monopolistic competition"), needs to be considered in order to derive more plausible models of the pass-through. Indeed, this also makes from the pass-through modelling an important part of model set-up.

Degrees of pass-through and the CPI inflation targeting

Usually, monetary policy aims at stabilisation of the consumer price index (CPI) inflation. This is true, even though some theorists suggested to target other variables instead (and Ball (2000) discussed below is definitely one of them), effectively in order to avoid possible over-reaction to short-term exchange rate fluctuations. The CPI measure can indeed be broken down to the domestically produced goods and imported goods, and the direct exchange rate channel may in turn be defined as an effect of the change in domestic-currency price of imported final goods on the CPI.

[1] One may define the exchange rate pass-through more precisely as "the transmission of a change in import costs to domestic prices of imported goods", cf. Flamini (2001), p. 2.

Several points can be made about the link between the degree of pass-through and stabilisation of CPI inflation. Let us first consider the case of complete pass-through, which illustrates most of the following results. In general, the domestic currency price of imported foreign goods, p_t^f (in logs) can be written as:

$$p_t^f = p_t^* + s_t$$

Here, p_t^* is the log of foreign price, and s_t is indeed the log of nominal exchange rate. Under such a notation, inflation of imported foreign goods, $\pi_t^f \equiv p_t^f - p_{t-1}^f$, becomes:

$$\pi_t^f = \pi_t + q_t - q_{t-1}$$

Here, π_t is domestic inflation, and q_t is the log of real exchange rate. Now, the CPI inflation, π_t^{cpi}, can be shown to be equal to the weighted average of domestic and imported inflation. The weights are equivalent to the share of imported goods in the CPI (ω):

$$\pi_t^{cpi} = \pi_t + \omega(q_t - q_{t-1})$$

In the above equation, indeed, domestic inflation cannot be controlled easily if there are substantial lags in the aggregate demand and expectations channels. As this is usually the case, the only tool for the monetary policy in the short-run seems to be the direct exchange rate channel affecting CPI only through the fraction of imported foreign goods.

When the degree of pass-through decreases, the effect of this channel is weakened and monetary policy looses its short-term instrument. On the other hand, the CPI target gets more insulated against foreign shocks: changes in foreign inflation and foreign interest rate, for example.

Rules with exchange rate: The Case Against

Monetary policy rule with the exchange rate feedback can be thought of as:

$$i_t = f\pi_t + gy_t + h_0 e_t + h_1 e_{t-1} \quad (1)$$

Here, i_t is the short-term interest rate, π_t is the rate of inflation, y_t is the deviation of real output from the potential, and e_t is the real exchange rate. As there is no intercept in (1), the target rate of inflation is assumed zero, and, indeed, i_t and e_t are measured as deviations from their long-run steady-state values. (That, in practice, involves estimation of the real equilibrium exchange rate (REER), rather a controversial task of its own.)

Ball (1999) studies a version of the rule that has $h_0 = -0.37$ and $h_1 = 0.17$. As both h_0 and $h_0 + h_1$ are negative, coefficients are consistent with the "rule of thumb" according to which the real appreciation shall call for loosening of monetary policy (cf. Obstfeld-Rogoff (1995)). The negative contemporaneous response of interest rate to exchange rate, h_0, is given by the contractionary effect of appreciation on the aggregate demand. The partial interest rate offset in the next period, though, is "due to the lagged impact of the appreciated exchange rate on inflation".

Initially, the measured rate of inflation decreases with appreciation. As, however, this decline in inflation *is temporary*, it would be inappropriate for the central bank to ease monetary policy further along the equation (1). The positive coefficient h_1 prevents this additional excessive easing.

Simulation Results

The results from Ball (1999) model show very moderate improvement of the inflation variability when the rule with exchange rate is used instead of the Taylor rule. Standard deviation of inflation around its target decreases from 2.0 to 1.9, when the standard deviation of output around potential is kept constant at 1.4 per cent.

Svensson (2000) reports results from a different model, of a rule that has $h_0 = -0.45$ and $h_1 = 0.45$. Simulations from his model show that such a rule improves standard deviation of inflation (from 2.1 to 1.8 per cent), but is inferior to the Taylor rule in terms of output variability that rises from 1.7 to 1.8 per cent.

Explanation

We shall seek some explanation for such a negligible improvement from the rules with exchange rate. One is that given by Taylor (2000), which points out an important indirect reaction of interest rates to the exchange rate. Such an effect indeed rends a special feedback term in a policy rule unimportant, as a strong enough feedback can be present indirectly, through other terms in the rule-specifying equation.

Suppose then that the rule only reacts to inflation and real output. But as it is a contingency plan being used by the central bank over many periods, we may expect that future changes in inflation and output would lead to change in future interest rates. Now, consider a real appreciation that in many models would lower GDP (via expenditure switching toward imports) and lower inflation (as it slows down changes in imported goods prices).

These effects would, however, in reality occur with a lag. Because of this lag, appreciation of the exchange rate today *today* leads to lower output and inflation levels being expected *in the future*. With the rule in place, this increases the probability that the central bank will raise short-term interest rate in future. Assuming a rational expectations model of term structure, these expectations of lower future short-term rates will tend to lower long-term interest rates today. Hence, under the assumption of rational term structure, an appreciation would result in lowering the interest rates today, "even though the exchange rate is not directly at the policy rule"[2] (Taylor, 2000, p.8).

Indeed, the second point that Taylor (2000) mentions as possible explanation for negligible exchange rate role in the rule, is that there may be deviations of the exchange rate from the purchasing power parity (PPP) that should not be off-set, e.g. changes in productivity. Smoothing such changes out may in fact be counter-productive and the central bank shall avoid such an overly activist policy.

Rules with exchange rates: The Case In Favour

Ball (2000) discusses some extensions of his open-economy model (cf. Ball, 1999) and provides an explicit argument for the use of exchange rate term in monetary policy rule. The model used for analysis is virtually the same as in his (1999) paper:

$$y = \lambda y_{-1} - \beta r_{-1} - \delta e_{-1} + \varepsilon \qquad (1)$$

$$\pi = \pi_{-1} + \alpha y_{-1} - \gamma(e_{-1} - e_{-2}) + \eta \qquad (2)$$

$$e = \theta r + \nu \qquad (3)$$

Here, *r* is the real interest rate, *e* is the real exchange rate (and higher e means appreciation). All variables are in deviations from their equilibrium long-term values.

This underlying macroeconomic model has indeed to be completed for policy purposes by a specification of the target variable and by the rule for setting the instrument(s). Let us consider an appropriate target variable, and then move on to the instrument consideration in the next section.

Target for an Open Economy

Ball argues that central bank should target "long-term inflation", π^*, rather than current inflation. He defines the former as $\pi^* = \pi - \gamma e_{-1}$, hence as a measure of price level changes that filters out the transitory changes in real exchange rate. The reason for such a choice of

[2] This effect can be even strengthen when the central bank targets inflation and output forecasts. Hence, an appreciation lowers the forecasted values of output and inflation, and the bank would lower the short-term interest rate today.

target is that short-term deviations from the equilibrium real exchange rate are likely to be reversed in the future without any action taken by the central bank. In other words, π^* is the level of inflation that corresponds to long-term equilibrium that can be thought of as zero output gap.

More precisely, there are two reasons for targeting π^* rather than π. The first is inflation inertia. Equation (2) effectively claims that inflation follows a random walk with drift, where feedback terms on output and real exchange rate add to a drift term.

Hence, inflation inertia brings about permanent nature of past shocks to inflation. If inflation actually deviates from its target level, either due to economy overheating or adverse supply shocks, such deviations in inflation would persist indefinitely if monetary policy is accommodative. Since this propagated effect of past deviations would make inflation to drift away from the target, inertia actually forms a basis for policy actions (either tightening or loosing depending on whether a negative or positive shock to inflation actually occurs). The only exception to this is temporary change in the real exchange rate, since such a shock is expected to reverse itself in future. Temporary shocks of this kind are however properly filtered out by long-run inflation targeting.

Second reason for targeting long-run inflation is that it prevents too aggressive moves in exchange rate that may be associated with current inflation targeting. This danger stems from the fact that the fastest channel from monetary policy to inflation works through the exchange rate effect on import prices. Thus, the central bank may be tempted to use this channel excessively, and in order to keep current inflation close to its target, effectively move exchange rates around *too much*. Such excessive swings in exchange rate would eventually translate into large interest rate shifts and then cause unnecessary fluctuations in output. Long-run inflation targeting, by definition, avoids this problem, as the target itself is insulated from temporary exchange-rate movements and policymakers do not need to react to them at all.

The problem with long-run inflation targeting is obviously that one needs to distinguish between deviations from equilibrium (temporary change) and shifts in the equilibrium level itself (permanent change). On top of that, one needs to estimate the equilibrium real exchange rate together with the parameter γ (real exchange rate elasticity of output) in order to derive the appropriate target value, π^*. These are not easy tasks and many researchers actually proposed a simplification that would avoid this estimation. They argue that the inflation rate for prices of domestically produced goods may be an operationally successful approximation to the long-run inflation, since the domestic-price inflation is only negligibly affected by temporary exchange-rate fluctuations.

Instrument and Rule

It shall be clear by now that the question of target and the question of proper instrument are two *separate* issues that central bank needs to solve. The target specifies what developments in the economy shall form a basis for a policy action, while the rule for instrument setting explicitly recognises transmission channels that central bank deems to be important when any policy action is to be taken.

A precise form of the rule will depend on the model, and indeed, one can argue that the rule for an open economy should actually recognise both interest rate and exchange rate as its instruments. Hence, Ball (2000) argues in favour of a Monetary Conditions Index (MCI), which in turn can be translated into a monetary policy rule with exchange rate term:

$$wr + (1-w)e = ay + b\pi^{*} \quad (5)$$

$$r = \frac{a}{w}y + \frac{b}{w}\pi^{*} - \frac{1-w}{w}e \quad (6)$$

Here, the left-hand side of (5) is a Monetary Conditions Index, a weighted average of real interest rate and real exchange rate, which is deemed to capture an overall stance of monetary policy in a way that is superior to considering just the real interest rate.

The reason for such superiority is that taking exchange rate into account effectively amounts to admitting that should the central bank move the interest rate in response to any shock, there are (at least) two channels through which such change affects spending, and also (one period hence) inflation. Effects from both channels should be properly added up in order to get the correct overall impact of policy action.

We can describe the two effects more precisely by examining the model: increase in real interest rate (1) raises e by θ in the same period, and this in turn lowers y by $\delta\theta$ in the next period, (2) lowers y directly by β with the lag of one. This effect on spending in the next period translates into a downward pressure on inflation. In addition to that, there is a direct exchange-rate-to-current-inflation link, which is however filtered out by the long-run inflation targeting.

Thus, an appropriate monetary policy rule shall acknowledge that in the open economy, autonomous changes in exchange rate amount to deliberate changes in the interest rate (to some extent given by $\frac{\delta\theta}{\beta}$ ratio), and hence, shall incorporate an MCI as a measure of overall monetary policy stance. If the exchange rate is omitted in the rule, policy makers inevitably "miss opportunities to adjust interest rate to offset the effects of exchange rate on spending", and end up with "unnecessarily large fluctuations in inflation and output".[3]

Although in theory (5) can be easily inverted into (6) and both rules are hence equivalent, in practice, there are reasons why the choice between the two may matter. One reason for this may be the fact that central banks are often reluctant to change instruments by large amounts and/or reverse the direction of changes (increases and decreases in the rule) very often. This leads them to cautious reactions, often over many small steps in one direction. His is called instrument smoothing. Banks can smooth interest rate as well as MCI or any other instrument they are using.

For example, the interest rate smoothing is quite common. If we think of this as the special case of the instrument smoothing, we can show that MCI may actually provide an instrument that yields more optimal outcomes in comparison with the interest rate. To see this, consider a negative shock to the exchange rate (i.e., $\theta < 0$, in Equation (3)). Such a shock requires higher interest rate under both (5) and (6). But while the response under (5) is in fact dampened by smoothing, full interest rate response under (6) goes through without any change in MCI. Hence, there is no smoothing and monetary policy is appropriately tight with an MCI instrument.

3. Econometric Methods and Modelling

(This section is highly descriptive and only sums up the basic text book theory and few examples. It is rather a memorizing tool for the author than an organic and justifiable part of the paper.

Any reader familiar with basic econometrics shall probably skip it in its entirety.)

In this section, we deal with some issues related to the econometric part of the research. The choice of an appropriate modelling approach and its modifications given the non-stationary nature of the real-world time series are the main concerns here.

Systems of Equations vs. VAR Models

Systems of equations (Simultaneous Equations Models) are being used in the presence of *endogeneity* that renders the single equation techniques inappropriate. There are two different approaches to the estimation of identified systems: (1) limited information estimation, and (2)

---

[3] Ball (2000), p.8.

full information estimation. The first involves individual estimation of each equation in the model, while it provides for the simultaneous linkages among them. The second approach involves estimation of all equations at the same time.

Vector Auto-Regression (VAR) model is, in principle, an unrestricted model that yields the reduced-form estimates of joint effects that arise among endogenous variables in the model. Hence, VAR provides a good tool for determining the overall outcomes of all variables that are merged together in the vector, while these effects may either be restricted to lagged values of variables, or incorporate contemporaneous effects as well. Hence, in general, there is no reason why one should be able to recover the structural-form equations from an estimated VAR model. A simple example of under-identified model shows, that this may indeed be a case. Consider a bivariate system in the structural form:

$$\begin{aligned} y_t &= b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt} \\ z_t &= b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt} \end{aligned} \tag{13}$$

where both y_t and z_t are stationary, I(0) variables, and errors are white noise disturbances. Due to the contemporaneous effect of both y_t on z_t, and z_t on y_t, the system cannot be recovered in its structural form (13) when estimated as a VAR in the reduced form. Equations estimated by VAR are given by:

$$\begin{aligned} y_t &= a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + e_{1t} \\ z_t &= a_{20} + a_{21} y_t + a_{22} z_{t-1} + e_{2t} \end{aligned} \tag{14}$$

Here, the error terms e_{1t}, e_{2t} are now composites of both $\varepsilon_{yt}, \varepsilon_{zt}$:

$$\begin{aligned} e_{1t} &= \frac{\varepsilon_{yt} - b_{12}\varepsilon_{zt}}{1 - b_{12} b_{21}}, \\ e_{2t} &= \frac{\varepsilon_{zt} - b_{21}\varepsilon_{yt}}{1 - b_{12} b_{21}} \end{aligned} \tag{15}$$

Taking the variance of original shocks and composite error terms also into account, we end up with a structural-form model with 10 parameters, while a VAR estimation only yields 9 parameters. Hence, if we want to recover the original model structure from the estimated VAR model, we need to impose at least one restriction on the structural-form system.[4]

This issue is often referred to as a problem of identification. Indeed, in practice, the restrictions that we may decide to impose on a basis of theory, may not capture the underlying

[4] Note that the model reported in the appendix as well as the model taken from Ball (2000) may be thought of as structural form models derived from the economic theory.

data generating process and hence result in misspecification. Secondly, under-identified models cannot be easily linked back to an economic theory that is mostly formulated in terms of structural models. While sticking to the reduced-form diminishes the risk of misspecification, proper identification of the model is likely to yield vastly superior insights in terms of economic theory. Hence, we shall restrict ourselves to models that are identified, so that we can always recover the original structural form of the model.

This discussion also marks out the main difference between the systems of equations and (unrestricted) VAR models. As we have seen, the crucial difference is that systems of equations deal with structural-form relationships among variables, while VAR models are in general reduced-form models.

Cointegration in Single Equations and in Multivariate Systems

A conceptually straightforward generalization of the single equation case gives its multivariate counterpart. In order to keep the analysis tractable, let us start with the single equation framework and then just mention the appropriate multivariate results. In this, we draw substantially upon Harris (1995).

There are at least two ways of tackling cointegration in single equations. First approach, often referred to as the Engle-Granger two-step procedure, is based on (1) estimating a static long-run relationship using OLS: $y_t = \beta_0 + \beta_1 x_t + u_t$, (2) testing whether residuals $\hat{u}_t$ are stationary, i.e. $u_t \sim I(0)$. The idea behind this is that due to superconsistency the estimate of β is unbiased if $\hat{u}_t$ are stationary and hence, cointegration exists between y_t and x_t. However, there are some other problems, for example a common factor restriction that is effectively imposed in (2), and unnecessary exclusion of all short-run dynamics that may be present in the model.

Because of these limitations, a dynamic modelling procedure is a preferable alternative as it gives more powerful tests for cointegration, better estimates of long-run coefficients and better statistics for hypotheses testing. Dynamic modelling involves estimation of an autoregressive distributed lag (ADL) model: $A(L)y_t = B(L)x_t + u_t$, which can be rewritten in the equilibrium correction form.

However, there are two other assumptions that must be met by the data. First, we need all right-hand side variables x_t to be weakly exogenous for y_t. If this assumption is violated, a single equation model is inefficient in comparison with multivariate modelling. Secondly, if more than two variables are included in the equation, there may be more than one

cointegrating relationship (actually, for *n* variables, there can exist up to *n-1* linearly independent cointegrating vectors). If more than one such relationship exists, single equation model can only give us a linear combination of them as the outcome of estimation. As it is unable to distinguish among them, the appropriate strategy is then multivariate modelling.
In general, cointegration analysis can be seen as an attempt to provide a reasonable model from the ADL class of models that is at the same time *invertible* into an equilibrium correction model (ECM) form:

$$\Delta y_t = \beta_0 \Delta x_t + (\alpha_1 - 1)(y_{t-1} - Kx_{t-1}) + \varepsilon_t \tag{9}$$

Here, $K = \dfrac{\beta_0 + \beta_1}{1-\alpha_1}$ is the long-run response coefficient (well-defined if $\alpha_1 \neq 1$), and the term $(y_{t-1} - Kx_{t-1})$ is called an equilibrium correction. The coefficient β_0 in the equation measures the rate at which contemporaneous changes in x_t occurs when y_t changes. The coefficient (α_1 -1) can be loosely interpreted as a measure of the speed of adjustment back to equilibrium if there is any shock that produces initial disequilibrium.

Interestingly, ECM models can be interpreted in terms of cointegration: There exists an isomorphic relationship between ECM and cointegrating processes, ensuring that variables that are cointegrated can be expressed uniquely in terms of ECM model. If y_t, x_t are both *I(1)* and cointegrated, then there exists an equilibrium correction mechanism of the form *(y-Kx)* **and conversely** [cf. Doornik and Hendry (2001)][5].

Now, we may extend the analysis into a multivariate setting. Similarly to the single equation ADL model, one can think of modelling a vector z_t of *n* potentially endogenous variables as an unrestricted VAR, incorporating, say, *k* lags of z_t:

$$z_t = A_1 z_{t-1} + ... + A_k z_{t-k} + u_t \tag{10}$$

Here z_t is a *(nx1)* vector of endogenous (simultaneously determined) variables, and A_i is a constant *(nxn)* matrix of coefficients. This is a reduced-form model, and as each variable in z_t is regressed on only lagged (and thus predetermined) variables, the OLS can be used for estimating each equation in (10) separately.[6]

[5] This is in fact a basic result in the theory of cointegration. It is called the *Granger representation theorem*, established by Engle and Granger (1987).

[6] In a way, an example we use here is a bit oversimplified, as many open economy models in fact assume contemporaneous effects among variables in z_t. In Ball (2000), for example, real exchange rate is assumed to react contemporaneously to real interest rate changes. Hence, OLS estimation is not appropriate in such cases.

As we know from the representation theorem, under the assumption of cointegration the model (10) can be re-formulated as a vector error-correction model (VECM):

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + ... + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + u_t \qquad (11)$$

Where $\Gamma_i = -\left(I - A_1 - ... - A_i\right)$, *for i=1,…,k-1*, and $\Pi = -\left(I - A_1 - ... - A_k\right)$. In fact, the matrix contains the key to long-run relationships among variables, and can be factorised as: $\Pi = \alpha\beta'$. The matrix α then gives the speed of adjustment to disequilibrium, while matrix β sums up the long-run coefficients and hence determines a cointegrating behaviour of the system.

The only point we shall make here, is that the method only works simply with variables that are either *I(0)* or *I(1)*. If there are variables of higher order of integration in the model, we must modify the approach along the lines outlined in Johansen (1994). As the Czech Republic data do not seem to involve *I(2)* variables, we do not need to explore this approach here any further.

Benchmark Modelling Strategy

The basic steps involved in estimating a cointegrated VAR model can be summarised in terms of an approach taken by Johansen and Juselius (1992). Here, we draw upon the summary provided in Harris (1995), which enumerates following seven steps:

1. Testing the order of integration of each variable that enters the multivariate model.
2. Decision about trends in the data, which determines whether or not the deterministic variables should be restricted to enter the cointegration space.
3. The lag-length of the VAR model that ensures Gaussian error terms in the VECM. This involves an issue of conditioning upon any predetermined *I(0)* variables (dummies for policy interventions, etc.)
4. Testing for a reduced rank.
5. Testing for weak exogeneity which leads to the modelling of a partial system with exogenous variables.
6. Testing for linear hypothesis on cointegration relations.
7. Joint tests of restrictions on α and β.

Appendix 1: Unit-root tests for the Czech data

In this section, I briefly report results of unit-root tests obtained so far for the Czech Republic time-series. The testing procedure is based on tests incorporated into the PC Give software package, and involves effectively Augmented Dickey-Fuller (ADF) tests. No other types of unit-root tests were conducted.

The results indicate that none of the series is in fact I(2) process. Among the series, I(0) variables – stationarity cannot be rejected – are:

- LRER (log of the real exchange rate), with constant and trend, at lag 2,
- INFL (inflation defined as a year on year change in CPI prices), no deterministic terms, with lag 4.

Non-stationary, I(1) variables are:

- LPRICES (log of CPI prices), with constant, at 0 lag,
- LNER (log of nominal effective exchange rate), with constant, at 0 lag.

One should note that cointegrating models (as VECM) provide a basis for alternative testing for unit roots. Hence, I shall set up a multivariate cointegrating model in line with these preliminary findings and then test for the appropriate restrictions within this framework.

Unit-root tests for 1993 (2) to 2000 (4)

INFL: augmented Dickey-Fuller tests (T=31)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-2.931**	0.91194	0.01750	-3.173	0.0039	-7.944	
3	-2.335*	0.91916	0.02023	-1.022	0.3160	-7.682	0.0039
2	-2.539*	0.91326	0.02024	0.1884	0.8519	-7.708	0.0087
1	-2.591*	0.91443	0.01990	1.776	0.0862	-7.771	0.0213
0	-2.421*	0.91732	0.02060			-7.733	0.0131

This suggests inflation is I(0), lag being 4. Constant (and hence the mean is zero).
However, INFL = PRICES_4, not simply DPRICES.

LPRICES: augmented Dickey-Fuller tests (T=35, Seasonals)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-3.056**	0.95177	0.01587	-1.001	0.3259	-8.089	
3	-2.904**	0.95840	0.01587	-0.08098	0.9360	-8.110	0.3259
2	-3.011**	0.95866	0.01559	-0.8382	0.4088	-8.167	0.6097
1	-2.908**	0.96159	0.01552	0.9726	0.3385	-8.200	0.6446
0	-3.740**	0.95584	0.01550			-8.226	0.6333

Even I(0) with seasonals?

Unit-root tests for 1992 (3) to 2000 (4)

DLPRICES: augmented Dickey-Fuller tests (T=34)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-0.9966	0.85694	0.02095	0.6288	0.5344	-7.597	

3	-0.9765	0.86141	0.02073	-2.519	0.0174	-7.642	0.5344
2	-1.512	0.77459	0.02245	-0.8307	0.4125	-7.509	0.0509
1	-1.825	0.74005	0.02234	-2.066	0.0470	-7.546	0.0820
0	-2.815**	0.61768	0.02342			-7.479	0.0320

Unit-root tests for 1992 (3) to 2000 (4)

DLPRICES: augmented Dickey-Fuller tests (T=34, Constant)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-2.046	0.34699	0.02021	1.231	0.2285	-7.644	
3	-1.718	0.47881	0.02039	-1.726	0.0950	-7.650	0.2285
2	-2.793	0.23019	0.02105	0.2734	0.7864	-7.611	0.1217
1	-3.158*	0.26966	0.02074	-0.5405	0.5927	-7.668	0.2254
0	-4.624**	0.18939	0.02051			-7.717	0.3192

So, I(1) with constant.

Unit-root tests for 1995 (3) to 2000 (3)

DLY: augmented Dickey-Fuller tests (T=21, Seasonals)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-1.399	0.60229	0.009869	-1.052	0.3118	-8.954	
3	-1.917	0.49170	0.009907	1.569	0.1389	-8.968	0.3118
2	-1.580	0.56868	0.01038	-0.8699	0.3981	-8.901	0.2050
1	-1.891	0.50597	0.01030	-1.079	0.2965	-8.947	0.2672
0	-3.016**	0.34671	0.01035			-8.972	0.2810

Suggest that LY is I(1) with seasonals.

Unit-root tests for 1994 (2) to 2000 (4)

LRER: augmented Dickey-Fuller tests (T=27, Constant, Trend)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-2.908	-0.13717	0.03009	0.5143	0.6127	-6.788	
3	-3.274*	-0.019380	0.02956	0.4954	0.6254	-6.849	0.6127
2	-3.973**	0.081711	0.02905	2.563	0.0177	-6.912	0.7807
1	-2.763	0.39920	0.03238	0.6515	0.5212	-6.725	0.1187
0	-2.894	0.47517	0.03199			-6.780	0.1727

Seems to be I(0)with lag 2.

Unit-root tests for 1994 (3) to 2000 (4)

DLNER: augmented Dickey-Fuller tests (T=26)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-2.821**	-0.39882	0.03702	0.5018	0.6210	-6.422	
3	-3.024**	-0.27358	0.03638	0.3986	0.6940	-6.487	0.6210
2	-3.456**	-0.17874	0.03571	1.201	0.2420	-6.557	0.8182
1	-3.322**	0.048073	0.03604	-0.1112	0.9124	-6.573	0.6331
0	-4.834**	0.025877	0.03532			-6.649	0.7785

Unit-root tests for 1994 (3) to 2000 (4)

DLNER: augmented Dickey-Fuller tests (T=26, Constant)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
4	-3.235*	-0.77096	0.03608	0.9250	0.3660	-6.445	
3	-3.299*	-0.48510	0.03596	0.7078	0.4868	-6.480	0.3660
2	-3.642*	-0.29294	0.03555	1.394	0.1772	-6.533	0.5196
1	-3.403*	-0.0079544	0.03627	0.02993	0.9764	-6.525	0.3802
0	-4.883**	-0.0016871	0.03551			-6.602	0.5334

So, I(0) variables: INFL, LRER,
I(1) variables: LPRICES, LY, LNER

Common view in the literature[7] dealing with Western European data, however, has that LPRICES is usually an I(2) process, implying INFL is I(1). LRER is either I(0) or I(1) in the datasets. These inconsistencies may be attributed to low power of the tests and we shall corroborate the findings within the multivariate model.

The data the tests were performed upon are graphed in an attached file:

[7] Cf. Harris (1995), Chap. 2.

Appendix 2: The Model

The model assumes that all imports are used as material inputs in domestic production. The equations of the model can be summarised as follows:

$$y_t = \alpha_0 + \alpha_1.y_{t-1} + \alpha_2.E_t y_{t+1} + \alpha_3.[R_t - E_t \Delta p_{t+1}] + \alpha_4.(\eta.q_t + y_t^*) \quad (1)$$

$$\Delta p_t = \beta.[\gamma_1.\Delta p_{t-1} + \gamma_2.E_t \Delta p_{t+1}] + \gamma_3 (y_t - \bar{y}_t) \quad (2)$$

$$R_t - R_t^* = E_t \Delta s_{t+1} + \xi_t \quad (3)$$

$$\Delta s_t = \Delta q_t + \Delta p_t \quad (4)$$

$$R_t = g_0 + g_1.E_t \Delta p_{t+1} + g_2.(y_t - \bar{y}_t) + g_3.s_t \quad (5)$$

As for the notation, y_t is the real domestic output, y_t^* is the real output abroad (world level), Δp_t is a measure of inflation, q_t is the price of imports in terms of consumption goods, $Q_t = \ln q_t$ being the real exchange rate. R_t is domestic one period nominal interest rate, while R_t^* is the appropriate foreign nominal interest rate (world level). In addition, the potential output is denoted $\bar{y}_t$, the p_t, p_t^* are logs of home and foreign prices of imports respectively, and S_t is the nominal exchange rate. The error term ξ_t can be interpreted as a time-varying risk premium and can explain pertaining deviations of the real exchange rate from the level implied by the uncovered interest rate parity condition.
Equation (1) is an aggregate demand relationship (an IS curve), (2) is a pricing-relationship (aggregate supply), (3) is an uncovered interest parity condition, (4) is a real exchange rate identity, and (5) is a policy rule.

Potential output is not explicitly modelled - I will use simple deviation from the logarithmic trend over the sample period as a measure of the potential output.

Optimizing framework of the model

The following is based on McCallum-Nelson (1999) which presents the original optimizing framework used in McCallum-Nelson (2000)

1. Aggregate demand specification

Economy consists of a continuum of households over (0,1) and each of them *maximize*

$$E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}, C_{t+j-1}, M_{t+j} / P_{t+j}^A)$$

where C is an index of household consumption in time t, M/P^A is the end-of-period real money holdings, and P^A is a general price level.
Utility function *u(.)* is separable across consumption and money balances:

$$u(C_t, C_{t-1}, M_t / P_t^A) = \exp(v_t) \frac{\sigma}{\sigma - 1} \left(\frac{C_t}{C_{t-1}^h} \right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{1-\gamma} \left(\frac{M_t}{P_t^A} \right)^{1-\gamma}$$

sigma, gamma positive, <>1, $h \in [0,1)$. v is a preference shock. This incorporates habit formation (through non-zero h coefficient).

Each household consumes many goods, *all of them domestically produced*. C(t) is a quantity of an index of goods constructed in Dixit, Stiglitz (1977) manner.

However, each household specializes in production, using the CES technology:

$$Y_t = \left[\alpha_1 (A_t N_t)^{v_1} + (1-\alpha_1)(IM_t)^{v_1}\right]^{\frac{1}{v_1}} \qquad (5)$$

alfa$\in$(0,1], *A(t)* being exogenous technological shock, *N(t)* labour hired in *t*, *IM(t)* foreign goods purchased <u>as an input in production</u>. Household exercises monopoly power over the price of good it produces, P_t, while aggregate price level, P_t^A ,foreign price level, P_t^* , and nominal exchange rate, S_t, is taken by the household as given.
Household either sells its output to domestic buyers or to the rest of the world, without any price discrimination: $Y_t = D_t + EX_t$.

Obstfeld, Rogoff (1996, Chap.10) showed that in such a setting:

$$D_t = \left(\frac{P_t}{P_t^A}\right)^{-\theta} D_t^A \qquad (6)$$

Assume, that foreign demand for domestic output is given accordingly:

$$EX_t = \left(\frac{P_t}{P_t^A}\right)^{-\theta} EX_t^A \qquad (7)$$

Note, that exchange rate does not appear in this, because it cancels from the relative price term on the right-hand side. Total demand for exports is in turn given by:

$$EX_t^A = \left(\frac{S_t P_t^*}{P_t^A}\right)^{\eta} Y_t^{*b}$$

with *eta*, *b* positive. Aggregate export demand is thus positively related to real exchange rate, $Q_t = \frac{S_t P_t^*}{P_t^A}$.

There are no bonds issued by government, but both domestic and foreign privately issued securities, denominated in units of domestic and foreign output respectively. Domestic agents can buy foreign bond at price $(1+\kappa_t)^{-1}(1+r_t^*)^{-1}$, where κ_t reflects temporary but persistent shifts from the uncovered interest parity condition. Such a bond is redeemed for one unit of foreign output next period. Similar relationship holds for domestic bonds that cannot be purchased by foreigners.

Let B_{t+1}, B_{t+1}^* denote quantities of domestic and foreign bonds respectively, purchased in t by a representative household. Then, budget constraint for a typical household in real terms becomes:

$$(\frac{P_t}{P_t^A})D_t + (\frac{P_t}{P_t^A})EX_t - C_t + (\frac{W_t}{P_t^A})N_t^S - (\frac{W_t}{P_t^A})N_t - (\frac{M_t}{P_t^A}) + (\frac{M_{t-1}}{P_t^A}) - B_{t+1}(1+r_t)^{-1} + B_t$$
$$- Q_t IM_t - Q_t B_{t+1}^*(1+\kappa_t)^{-1}(1+r_t^*)^{-1} + Q_t B_t^* = 0 \quad (9)$$

where W_t is nominal wage and N_t^S is a household labour supply. Letting ξ_t and λ_t be a Lagrange multiplier on (5) and (9) respectively, we obtain following first order conditions:

$$\exp(v_t)(\frac{1}{C_{t-1}^h})^{\frac{\sigma-1}{\sigma}} C_t^{-\frac{1}{\sigma}} - \beta h E_t \exp(v_{t+1}) C_t^{\frac{h-\sigma h-\sigma}{\sigma}} C_{t+1}^{\frac{\sigma-1}{\sigma}} = \lambda_t \quad (10)$$

$$(\frac{M_t}{P_t^A})^{-\gamma} + \lambda_t E_t\left[(1+r_t)^{-1}\frac{P_t^A}{P_{t+1}^A} - 1\right] = 0 \quad (11)$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1+r_t) \quad (12)$$

$$Q_t \lambda_t = \beta E_t Q_{t+1}\lambda_{t+1}(1+\kappa_t)(1+r_t^*) \quad (13)$$

As household also chooses optimal amount of labour and imported inputs into production, we obtain two additional conditions:

$$(\frac{\lambda_t}{\xi_t} \cdot \frac{W_t}{P_t^A})^{\frac{1}{1-v_1}} = a_1^{\frac{1}{1-V_1}} A_t^{\frac{v_1}{1-v_1}} \cdot \frac{Y_t}{N_t} \quad (14)$$

$$(\frac{\lambda_t}{\xi_t} . Q_t)^{\frac{1}{1-v_1}} = (1-a_1)^{\frac{1}{1-v_1}} \frac{Y_t}{IM_t} \quad (15)$$

If we define domestic and foreign (starred) interest rates in a Fischer relationship-like fashion, from (12),(13) follows the UIP relationship:

$$R_t = R_t^* + E_t \Delta s_{t+1} + \kappa_t \quad (16)$$

where $s_t = \log S_t$.

2. <u>Price Adjustment</u>

Gradual adjustment is considered in this framework. Taking logs of (6), (7), with lower case letters denoting logs of upper cases:

$$d_t = d_t^A - \theta(p_t - p_t^A)$$

$$ex_t = ex_t^A - \theta(p_t - p_t^A)$$

Since all product is sold either at home or abroad, making log-linear approximation we obtain:

$$y_t = (1 - EX^{ss}/Y^{ss})d_t + (EX^{ss}/Y^{ss})ex_t$$

which also holds for aggregate values of *y*, *d*, and *ex* (since the equilibrium is symmetric). Implied in this is:

$$y_t - y_t^A = -\theta(p_t - p_t^A)$$

which also hold for flexible prices values of $\bar{y}_t, \bar{p}_t$ as well. Without adjustment costs, $\bar{p}_t$ would be an optimal choice. Here, however, we suppose price-setting one period in advance, with adjustment costs. Thus, the problem becomes to *minimize*:

$$E_{t-1}\sum_{j=0}^{\infty}\beta^j\left\{(p_{t+j}-\bar{p}_{t+j})^2+\gamma_1\left(y_{t+j}-\bar{y}_{t+j}-\left[y_{t+j-1}-\bar{y}_{t+j-1}\right]\right)^2\right\} \tag{21}$$

Solving the Euler equation for (21) yields the decision rule for *p(t)*, *y(t)*:

$$E_{t-1}\tilde{p}_t = \phi.\tilde{p}_{t-1}$$

and $$E_{t-1}\tilde{y}_t = \phi.\tilde{y}_{t-1} \tag{25}$$

where tilded variables denote deviations from flexible price values $\bar{y}_t, \bar{p}_t$. Moreover, in symmetric equilibrium, this individual equation will also hold for the aggregate output gap. Equation (25) also reveals that strict version of the natural rate hypothesis holds in this model.

Now, recalling (5), we can define flexible price output, $\bar{Y}_t$. Since under price flexibility $N_t = N_t^S = 1$ for all t, (5) becomes:

$$\bar{Y}_t = \left[\alpha_1(A_t)^{v_1} + (1-\alpha_1)(IM_t)^{v_1}\right]^{\frac{1}{v_1}} \tag{27}$$

and using log-linear approximation:

$$\bar{y}_t = (1-\delta)a_t + \delta.\overline{im}_t \tag{28}$$

where: $\delta \equiv (1-\alpha_1)\left(\overline{IM}^{ss}/\bar{Y}^{ss}\right)^{v_1} = \dfrac{\theta}{\theta-1}\dfrac{Q^{ss}.IM^{ss}}{Y^{ss}}$.

From (15), letting $q_t \equiv \log Q_t$, and omitting the intercept, we get:

$$im_t = y_t - \frac{1}{1-v_1}.\log(\frac{\lambda_t}{\xi_t}) - \frac{1}{1-v_1}.q_t + \frac{1}{1-v_1}.\log(1-\alpha_1) \tag{29}$$

Under price flexibility, markup is constant $(\lambda_t/\xi_t) = \dfrac{\theta}{\theta-1}$ and (29) imply, omitting the constant once again:

$$\overline{im}_t = \bar{y}_t - \frac{1}{1-v_1}.q_t \tag{30}$$

Substituting from (30) into (28), we can see that:

$$\bar{y}_t = a_t - \omega . q_t \qquad \text{where: } \omega \equiv \frac{\delta}{(1-v_1)(1-\delta)}.$$

Flexible level of output is a function of the technological shock and the real exchange rate. This relationship capture the route by which exchange rate changes that are reflected in RER changes affect the price of domestic goods.

Pricing mechanism is very similar to one of the P-bar model, McCallum (1994), which implies that p_t is set according to $E_{t-1}\bar{p}_t$. Thus, changes in s_t that affect q_t lead after one period to rapid changes in p_t.

3. Log Linearization

Log linearizing (10) and (12) we obtain in the end expectational differential equation for the change in consumption:

$$\begin{aligned} &\beta.(h-\sigma h)E_t\Delta c_{t+2} + (1+\beta h^2 - \sigma\beta h^2 - \sigma\beta h)E_t\Delta c_{t+1} + \sigma(1-\beta h)E_t\Delta p_{t+1} = \\ &= (h-\sigma h)\Delta c_t + \sigma(1-\beta h)R_t - \sigma(1+\rho_v - \beta h\rho_v^2 + \beta h\rho_v)v_t \end{aligned} \tag{34}$$

The other equations in the log linearized model include (16), (25), (30), (31) and:

$$y_t = (C^{ss}/Y^{ss})c_t + (EX^{ss}/Y^{ss})ex_t \tag{35}$$

When completed by a policy rule for R_t, this model is a linear rational expectations system which can be solved by Klein`s (1997) algorithm.

Structural model proposed for the exercise – as outlined in McCallum, Nelson (2000) – thus consists of the consumption condition:

$$c_t = E_t c_{t+1} + b_0 + b_1(R_t - E_t\Delta p_{t+1}) + v_t$$

the Calvo pricing equation (amended as in Fuhrer, More (1995)):

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \lambda(y_t - \bar{y}_t)$$

the definition of potential output:

$$\bar{y}_t = a_t - [\sigma\alpha/(1-\alpha)].q_t + const.$$

UIP condition:

$$R_t - R_t^* = E_t\Delta s_{t+1} + \kappa_t$$

identity defining the real exchange rate:

$$\Delta s_t = \Delta q_t + \Delta p_t$$

and the following version of aggregate demand equation (amended as in McCallum (2000)):

$$y_t = \varpi_1 c_t + (1-\varpi_1)\eta . q_t + (1-\varpi_1) b . y_t^*$$

Model is to be completed by an appropriate specification of monetary policy rule.

Bibliography

Ball, L. (1999). Policy Rules for Open Economies. Monetary Policy Rules. J. B. Taylor. Chicago, University of Chicago Press.

Ball, L. (2000). Policy Rules and External Shocks. Working Paper. Johns Hopkins University**:** 20 p.

Batini, N. and A. Haldane (1999). Forward-looking rules for monetary policy. Monetary Policy Rules. J. B. Taylor. Chicago, University of Chicago Press.

Burstein, A., M. Eichenbaum, et al. (2001). Why Are Rates of Inflation So Low After Large Contractionary Devaluations? Northwestern University Working Paper**:** 51 p.

Calvo, G. A. (1983). "Staggered Prices in a Utility-Maximizing Framework." Journal of Monetary Economics **12**(3): 983-998.

Enders, W. (1995). Applied Econometric Time Series. New York, John Wiley.

Flamini, A. (2001). Open-Economy Inflation Targeting and Exchange Rate Pass-Through. HEI Working Paper. Geneva**:** 22 p.

Fuhrer, J. C. and G. R. Moore (1995). "Inflation Persistence." Quarterly Journal of Economics **110**(1): 127-159.

Harris, R. (1995). Using Cointegration Analysis in Econometric Modelling. Hemel Hempstead, Prentice Hall/Harvester Wheatsheaf.

Hendry, D. F. and J. A. Doornik (2001). Empirical Econometric Modelling Using Pc Give 10. London, Timberlake Consultants Press.

Johansen, S. (1994). "A statistical analysis of I(2) variables." Econometric Theory.

Johansen, S. and K. Juselius (1992). "Testing structural hypotheses in a multivariate cointegration analysis of the PPP and UIP for UK." Journal of Econometrics(53): 211-244.

McCallum, B. T. (1998). "Solutions to Linear Rational Expectations Model: A Compact Exposition." Economic Letters **61**: 143-147.

McCallum, B. T. and E. Nelson (2000). "Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices." Oxford Review of Economic Policy **16**(4).

Obstfeld, M. and K. Rogoff (1995). "The Mirage of Fixed Exchange Rates." Journal of Economic Perspectives **9**(4): 73-96.

Svensson, L. E. O. (2000). "Open-Economy Inflation Targeting." Journal of International Economics **50**: 155-183.

Taylor, J. B. (1993). Macroeconomic policy in a world economy: From econometric design to practical operation. New York, W. W. Norton.

Taylor, J. B. (2000a). "Alternative Views of the Monetary Transmission Mechanism: What Difference Do They Make for Monetary Policy?" Oxford Review of Economic Policy **16**(4).

Taylor, J. B. (2000b). The Role of the Exchange Rate in Monetary Policy Rules.

Taylor, J. B. (2001). "Low inflation, pass-through, and the pricing power of firms." European Economic Review: forthcoming.

Uhlig, H. (1997). A Toolkit for Analyzing Non-linear Dynamic Stochastic Systems Easily. Working Paper. University of Tilburg.